Reliability of Marine Structures Program

CYCLES 2.0:

FATIGUE RELIABILITY MODELS AND RESULTS FOR WAVE AND WIND APPLICATIONS

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Department of CIVIL ENGINEERING
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Acknowledgments

The original CYCLES code (version 1.0) was developed by Clifford H. Lange during the course of his Ph.D. studies within the Reliability of Marine Structure Program at Stanford University. This original version was documented in RMS Report No. 13, together with an associated PC-executable and the source code of the fatigue limit state function.

This document describes a more general fatigue formulation, and its implementation, within CYCLES version 2.0. It has been developed within the course of the Ph.D. studies of the first author, Alok K. Jha. These generalizations were particularly motivated for applications to fatigue-sensitive components of ships or of wind turbine machines. These applications share the common features of a complex load environment, potential interactions between the structure and its environment, and hence the need to best utilize available loads data, from either limited test records or numerical simulations. Examples of both ship and wind turbine fatigue applications are shown in this report.

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Abstract

CYCLES is a computer program that computes the fatigue reliability of mechanical components. It includes a rather flexible model of uncertainty, both in distribution parameters of randomly varying quantities (e.g., load environment parameters such as wave height, wind speed, etc.) and in uncertain material properties (e.g., S–N fatigue properties). The formulation is intended to be of general applicability across a range of fatigue problems. Applications are shown here to offshore structures and wind turbines, both of which may experience fatigue problems.

These models are efficiently analyzed through FORM/SORM techniques (first- and second-order reliability methods). A simple, analytical g-function (limit state) is established, which may be directly incorporated into standard FORM/SORM software packages.

We describe here one such code, which includes a 24-variable formulation, its capabilities, input parameters, and output. Fatigue reliability results are shown from this code for both wave and wind applications, including failure probability variation across a range of target fatigue lives.

Chapter 1

Introduction and Background

Fatigue life estimates for structures and mechanical components are known to be highly variable, particularly at early stages of design. It may be unreasonable at this stage to seek a single precise estimate of fatigue life. Indeed, such an estimate might show considerable variability across a range of environmental and structural modelling assumptions. This suggests that any such number should be reported with an appropriate range of uncertainty, reflecting both natural uncertainty and our professional ignorance of precise structural behavior, mechanical fatigue laws, and so forth. This in turn suggests that the proper question may not be "what is the actual fatigue life of this component?", but rather "with what confidence will the component meet its target design life?" Such questions are naturally addressed by the theory of structural reliability.

We present a formulation of the fatigue damage problem that considers this question. It is by no means the most complex or detailed model that can be established, but we believe it represents a useful compromise between its level of detail in mechanical and probabilistic modelling, and our state of knowledge. It produces a convenient, analytical form of the limit-state ("g–function"), which can be directly analyzed through any of the various FORM/SORM computer codes that are currently available.

This fatigue formulation is intended to capture uncertainty in environmental loading, gross structural response, and local fatigue properties. Fatigue damage is modeled probabilistically using Miner's Rule and the effects of variable loads, mean stress effects, stress concentration factors, and uncertainty in the fatigue properties themselves are included. A critical distinction here is between continuously varying quantities such as an environmental parameter (e.g., significant wave height H_S, mean wind speed V, applied stress level S versus time, etc.) and fixed parameters which may be uncertain (e.g. fatigue law coefficients, distribution parameters of H_S, V, S given either H_S or V, etc.). Continuously varying quantities are reflected here implicitly, through their average effect on fatigue damage. In contrast, parameter uncertainty doesn't "average out" over fatigue life, and is modelled here explicitly.

This fatigue formulation is also intended to be of rather general applicability. It has first been developed (Veers, 1990) and since extended at Stanford with wind turbine applications in mind, but we believe it may be equally useful for offshore applications. Examples are shown here for both wave and wind applications.

The next chapter provides a detailed description of the formulation used to compute fatigue damage and therefore component lifetime. A brief description of the solution methods employed by the program is presented for completeness.

The remainder of this documentation describes the program CYCLES and its required input and output. Sample problems illustrating the program's capabilities are included. Appendices A and B contain the program output for the two sample problems discussed in Chapters 5 and 6, respectively.

1.1 Version history

The following is a list of the differences between versions 1.0 and 2.0 of CYCLES:

1. Version 2.0 of CYCLES permits the inclusion of two environment variables X_1 and X_2, as compared with only one environmental variable in Version 1. For marine structures, these dual attributes may be X_1=H_S and X_2=T_P, the significant wave height H_S and peak spectral period T_P of a steady-state seastate. For wind response we may instead choose X_1=V=mean wind speed, and X_2=I=turbulence intensity parameter.

2. In version 2.0, the cyclic stress amplitude S (given environment variables X_1 and X_2) can be modeled with any of a number of two-parameter distribution types, or as a three-parameter quadratic Weibull type which can match three moments of S given X_1 and X_2. Version 1.0 supported only a Weibull model for the conditional stress S, given the single environmental variable X_1.

3. New features have led to an increase in the number of random variables in Version 2.0 to 24, as compared with 14 random variables in Version 1.0.

4. The input format has been simplified in Version 2.0.

5. The distribution library of CYCLES 2.0 contains more distribution types. Additions in Version 2.0 include Exponential, Gumbel, and Quadratic Weibull models, as well as shifted versions of each of these.

6. Due to its additional complexity per run, CYCLES 2.0 does not automatically perform either (a) the sensitivity analyses or (b) the multiple target lifetimes within a single run, as did CYCLES 1.0. If desired, these quantities can be calculated through several runs of CYCLES 2.0.

Chapter 2

General Fatigue Formulation

Whether we consider fatigue or an alternate failure condition, a complete reliability formulation generally includes uncertainty in three distinct aspects:

1. The loading environment characterized here by random variables
2. The gross level of structural response given the load environment
3. The local failure criterion given both both the load environment and the gross stress response

For the fatigue limit state we examine each of these in turn below, along with the modelling capabilities CYCLES 2.0 affords for each.

2.1 Load Environment

Characterizing variables: X_1, X_2 = dominant environmental parameters

For the subsequent analysis, we assume the load environment is well characterized by two controlling random variables, herein denoted X_1 and X_2 for generality. We therefore require their joint probability density function, $f_{X_1,X_2}(x_1, x_2)$, as input to the fatigue reliability analysis. In this formulation we require the user to specify the marginal densities $f_{X_1}(x_1)$ and $f_{X_2}(x_2)$ of X_1 and X_2 and the correlation coefficient, ρ_{X_1,X_2}, between the two. These quantities can commonly be estimated from site-specific environmental data.

For offshore problems we may typically take $X_1{=}H_S$, the significant wave height and $X_2{=}T_P$, the peak spectral period during a period when the wave elevation process $\eta(t)$ can be assumed to be stationary (i.e., in a statistical steady-state condition). Following common convention, we define $H_S{=}4\sigma_\eta$, that is, 4 times the standard deviation (rms) of the wave elevation process. It is also roughly equal to the mean of the highest one-third of all wave heights (peak–trough distances), provided the common Gaussian model of $\eta(t)$ is assumed to hold. For wind, $X_1{=}V$, some measure of average wind speed and $X_2 = I$, some measure of turbulence intensity (ratio of rms to mean wind speed) over a reference period, and at a reference elevation.

In CYCLES 2.0, each of the marginal densities $f_{X_1}(x_1)$ and $f_{X_2}(x_2)$ can take on forms consistent with any of the two-parameter distribution types listed in Table 4.1. (Note that in principle, the methodology permits any distribution in the CYCLES library—whether it has two or more parameters—to be used to model X_1 and X_2. Our limitation here to two-parameter distribution types is to simplify not the method but rather the input. Because we assign two-parameter distribution types to each X_1 and X_2, we require only the mean value and the coefficient of variation (COV) of each of these variables.

Resulting uncertain quantities: $\overline{X_1}, V_{X_1}, \overline{X_2}, V_{X_2}$=mean and coefficient of variations of the environmental parameters X_1, X_2 (wave height and wave period, wind speed and turbulence intensity, etc.).

Resulting fixed parameters: The correlation coefficient ρ_{X_1,X_2} between X_1 and X_2; the distribution type indices `idistx1` and `idistx2` that define the distributions of X_1 and X_2, respectively, from the available choices in Table 4.1.

2.2 Gross Response

Characterizing Variable: S**=amplitude of local stress process.**

The stress response at the location of interest will typically not be regular (i.e., sinusoidal). Nonetheless, we assume that some method, such as rainflow counting, is available to identify amplitudes of equivalent stress "cycles." Statistics of an arbitrarily chosen amplitude S will generally depend on the underlying environmental variables X_1, X_2. Thus we generally require the conditional probability density $f_{S|X_1,X_2}(s|x_1,x_2)$, over all possible values of the environmental variables x_1, x_2.

Given the load environment X_1 and X_2, CYCLES 2.0 permits the stress amplitude S to be modeled with any one of the two-parameter distribution types listed in Table 4.1, or with a three-parameter quadratic Weibull model. These two-parameter distributions can generally preserve the first two moments of S, while the quadratic Weibull can be tuned to match rather general values of the first three moments of S. The moments of the stress are assumed to vary, given X_1=x_1 and X_2=x_2, in power-law fashion:

$$\mu_i(x_1,x_2) = a_{0i}\left(\frac{x_1}{x_{1_{\text{ref}}}}\right)^{a_{1i}}\left(\frac{x_2}{x_{2_{\text{ref}}}}\right)^{a_{2i}} \tag{2.1}$$

where μ_i, for $i = 1,2,3$, refers respectively to the mean, COV, and skewness of the stresses. The quantities $x_{1_{\text{ref}}}$ and $x_{2_{\text{ref}}}$ denote reference values for x_1 and x_2, chosen here for convenience as the geometric means of the data. For example, if x_1 has observed values $x_{11}...x_{1n}$ its geometric mean is defined as

$$x_{1_{\text{ref}}} = [x_{11}...x_{1n}]^{1/n} \tag{2.2}$$

The quantity $x_{2_{\text{ref}}}$ is defined similarly. Use of these reference values in Eqn. 2.1 ensures that linear regression, applied to the logarithm of this equation, yields estimates of a_{0i}, a_{1i}, and a_{2i} that are mutually uncorrelated. Similarly, the average stress cycle rate f_{avg} is assumed to follow a similar power-law variation with x_1 and x_2:

$$f_{avg}(x_1,x_2) = f_0\left(\frac{x_1}{x_{1_{\text{ref}}}}\right)^{f_1}\left(\frac{x_2}{x_{2_{\text{ref}}}}\right)^{f_2} \tag{2.3}$$

in terms of the same reference values $x_{1_{\text{ref}}}$ and $x_{2_{\text{ref}}}$, and the new coefficients f_0, f_1, and f_2.

The true values of these parameters, a_{ji} and f_j for terms $0 \leq j \leq 2$ and moments $1 \leq i \leq 3$, remain uncertain due to limited data. CYCLES 2.0 permits each of these 12 parameters to be modelled as random variables, whose means and standard deviations can be estimated from standard regression techniques. Any of the distribution types in Table 4.1 can be assigned in CYCLES 2.0 to each a_{ji} and each f_j.

Resulting uncertain quantities: Twelve values, estimating the 9 parameters a_{ji} and 3 parameters f_j ($0 \leq j \leq 2$ and $1 \leq i \leq 3$) arising in Eqn. 2.1 and Eqn. 2.3.

Resulting fixed parameters: The reference values $x_{1_{\mathrm{ref}}}$ and $x_{2_{\mathrm{ref}}}$, and the distribution index `idist_s` defining the distribution of $S|X_1, X_2$, from among any of the two-parameter distributions in CYCLES 2.0 or the three-parameter, quadratic Weibull model.

2.3 Failure Measure

Characterizing Variable: $\overline{D}$=mean value of Miner's damage.

We assume that fatigue tests at constant stress amplitude S are available to estimate the "S–N" curve; that is, the number of cycles $N_f(s)$ to failure as a function of stress amplitude s. Miner's rule is then used, assigning damage $1/N_f(S_i)$ due to a single stress at amplitude S_i. We assume here that this damage grows linearly at its mean rate $\overline{D}$, ignoring local variations in this rate due to variability in the cyclic amplitudes S_i. (This will tend to average out quickly for the high-cycle fatigue applications of interest here.) As a result, fatigue behavior is characterized by only the mean damage rate $\overline{D}$, and hence by only the "S–N" curve $N_f(S)$.

Specifically, CYCLES 2.0 takes the S–N curve as a straight line on log-log scale, with an effective intercept C_0 that includes the Goodman correction for mean stress effects:

$$N_f(S) = C\left(\frac{S}{1 - KS_m/S_u}\right)^{-b} = C_0 S^{-b}; \;\; C_0 = C(1 - KS_m/S_u)^b \tag{2.4}$$

in which S_m and S_u are the mean and ultimate stress levels.

Resulting uncertain quantities: Five parameters in general: the S-N parameters C and b, the stress concentration factor K, and the mean and ultimate stress levels, S_m and S_u. In practice it is common to fix b (and then estimate C from the data); alternatively, if C and b are both considered as uncertain, their estimates are generally correlated. (CYCLES 2.0 permits general correlation coefficients among these and other pairs of uncertain variables).

2.4 Limit State Evaluation

Characterizing Variable: Safety margin M=Excess time to fail beyond service life.

To summarize from the previous sections, the general fatigue formulation requires three functional inputs: $f_{X_1,X_2}(x_1,x_2)$, $f_{S|X_1,X_2}(s|x_1,x_2)$, and $N_f(s)$ to characterize the load, response, and fatigue damage respectively. A convenient scalar quantity on which to focus is the mean damage $\overline{D}$. This is found by integrating/summing over all load and response levels, x_1, x_2 and s:

$$\overline{D} = \int_{x_1=0}^{\infty}\int_{x_2=0}^{\infty}\int_{s=0}^{\infty} \frac{f_{avg}(x_1,x_2)}{N_f(s)} f_{S|X_1,X_2}(s|x_1,x_2) f_{X_1,X_2}(x_1,x_2) ds\, dx_1\, dx_2 \tag{2.5}$$

The average frequency, $f_{avg}(x_1,x_2)$ from Eqn. 2.3, is used here to convert the damage $1/N_f(S)$, per cycle, to a corresponding damage per unit time.

In general, Eqn. 2.5 can be evaluated numerically, permitting arbitrary functional forms for $f_{X_1,X_2}(x_1,x_2)$, $f_{S|X_1,X_2}(s|x_1,x_2)$, and $N_f(s)$. It is convenient (though not essential) to specialize here to the single power-law S–N curve, $N_f(S)$=C_0S^{-b}, from Eqn. 2.4. In this case Eqn. 2.5 can be rewritten as

$$\overline{D} = \frac{1}{C_0}\int_{x_1=0}^{\infty}\int_{x_2=0}^{\infty} f_{avg}(x_1,x_2) E[S^b|\, X_1=x_1,\ X_2=x_2] f_{X_1,X_2}(x_1,x_2) dx_1\, dx_2 \tag{2.6}$$

in which

$$E[S^b|\, X_1=x_1,\ X_2=x_2] = \int_{s=0}^{\infty} s^b f_{S|X_1,X_2}(s|x_1,x_2) ds \tag{2.7}$$

Once obtained, $\overline{D}$ can be used to directly estimate the fatigue life T_f. We assume that after the many cycles that contribute to high-cycle fatigue, the actual damage varies negligible from its average value $\overline{D}$. If we assume that failure occurs when this damage reaches a critical threshold Δ, and that the component experiences fatigue loads over a fraction A (availability) of its service life, its failure time is then estimated as

$$T_f = \frac{A\cdot\Delta}{\overline{D}} \tag{2.8}$$

If Miner's rule is considered exact we would assign Δ=1. More generally, variability in Δ would reflect inaccuracies in Miner's rule; i.e., the scatter resulting from predicting variable-amplitude fatigue lives from constant-amplitude tests. Finally, the fatigue safety margin M is defined as in Eqn. 3.1, repeated here for convenience:

$$M = T_f - T_t = \frac{A \cdot \Delta}{\overline{D}} - T_t \tag{2.9}$$

Resulting uncertain quantities: The damage threshold Δ, the availability fraction A, and the target service life T_t.

Resulting fixed parameters: The integer values **nquadx1** and **nquadx2**, which define the number of quadrature points used to evaluate the double integral in Eqn. 2.6. Specifically, that equation is evaluated by a double sum of the form

$$\overline{D} = \frac{1}{C_0} \sum_{i=1}^{nquadx1} \sum_{j=1}^{nquadx2} p_{ij} \cdot \left(f_{avg} E[S^b] \right) |x_1, x_2 = (\xi_1, \xi_2)_{ij} \tag{2.10}$$

in which $(\xi_1, \xi_2)_{ij}$ is the location of the i–j quadrature point approximating the continuous density $f(x_1, x_2)$, with associated probability weight p_{ij}. These points are obtained from standard Gaussian quadrature points for a standard normal variable U. If X_1 and X_2 are independent, each variable is separately related by a standard normal through its distribution function:

$$X_i = F_{X_i}^{-1}(U_i) \tag{2.11}$$

This relation is then applied at each Gauss quadrature point, to transform its location to reflect the marginal probability distribution of each X_i. If X_1 and X_2 are also correlated, a final step rotates the quadrature points of X_2 to reflect this trend in the bivariate density $f(x_1, x_2)$.

Chapter 3

Solution Algorithm for Failure Probability

For the reliability analysis the failure criterion is taken to be the difference between the computed fatigue life (Eqn. 2.8) and a specified target lifetime, T_t:

$$G(Y_1 ... Y_n) = T_f - T_t \tag{3.1}$$

Equation 3.1 is known as the limit state function, which when negative indicates inadequate structural performance ("failure"). The Y_i denote the set of all physical variables—here, n=24—that are uncertain (or random). These physical random variables are commonly denoted as X_i; here we choose Y_i to avoid confusion with the continuously varying environmental variables X_1 and X_2 defined earlier.

The solution for the failure probability is a three step procedure that is described here briefly for completeness. A more thorough description of reliability methods can be found in [7] and other references. The three steps are transformation, approximation, and computation.

The transformation requires that each physical random variable, Y_i, be associated with an uncorrelated, unit variance, normally distributed random variable U_i. For independent variables this is achieved by equating the cumulative distribution functions of the input variable and its associated standard normal variate (as in Eqn. 2.11 for the environmental variables X_1 and X_2). Correlation can be included by working with conditional distributions [7] Alternatively if only the marginal distributions and correlation coefficients among the Y_i are known, we may proceed in two steps:

- With conventional methods, each Y_i can be transformed marginally to a standard normal variable V_i. The resulting V_i variables will also be correlated, to a typically somewhat greater extent than the original physical (non-normal) variables x_i. Analytical methods have been developed to efficiently predict this correlation "distortion" due to non-normal physical variables [12].

- Correlation among the V_i's may be removed by standard methods (e.g., Cholesky decomposition of the covariance matrix) to obtain standard nomal variables U_i.

This is the approach used in CYCLES. All random variables are transformed in this fashion and the calculations proceed in standard normal space, also called "normal" or U–space.

The failure state function (eqn. 3.1) is evaluated in normal U–space and gradient search methods are employed to find where it is closest to the origin, also known as the design point. Approximation of the failure probability is obtained by fitting a tangent line (in the first order reliability method, or FORM) or a parabola (in the second order reliability method, or SORM) to the failure state function at the design point. The direction cosines of the vector that defines the design point are relative measures of the importance of each of the random variables.

The symmetry of standard normal space simplifies the computation of the failure probabilities and the importance factors. FORM probabilities are computed directly from the length of the vector identifying the design point. SORM estimates of failure probability are based upon the vector length and the curvatures of the surface at the design point.

Chapter 4

Program CYCLES

The current features of the CYCLES program are:

1. Calculation of mean excess life
2. First order (FORM) and second order (SORM) failure probabilities
3. Importance factors for each random variable
4. Option to run simulation

4.1 Capabilities

The primary result of the CYCLES program is an estimate of the "failure" probability, p_f, i.e. the probability that the fatigue life will be less than the component's target service life. It is estimated as described in the preceding section. The importance factors, which reflect the relative contribution of each variable to fatigue life uncertainty, are also reported.

4.2 Input Parameters

The program runs by default in the batch mode, in which input is read from the file "cycles.in". A sample input file is presented in Appendix A . This particular file is for a wind turbine application described in the example section. Each line is commented to provide a functional description of the variables contained on that line. The input is in free format, so the input file can contain the descriptive comments shown.

The first line contains default input and output file control parameters for code execution. Of special interest to the user is the logical unit number for subsequent input (IOIN). The value of IOIN=4, which is currently implemented, implies that the remaining input is read from "cycles.in". A log file, "cycles.log," is automatically generated each time the program is run.

The second and third lines of "cycles.in" contain the number of random variables, and number of fixed parameters respectively. These values do not need to be changed under the current implementation of the fatigue life problem. The current version of CYCLES runs with 14 fixed parameters and 24 random variables.

Lines 4 through 17 contain the values of the 14 fixed parameters. The first two are the reference values of the environment variables used in the power-law fits of the stress moments and the stress cycle rate. Line 6 and line 8 contain the distribution types for X_1 and X_2, respectively. Lines 7 and 9 contain the number of quadrature points for X_1 and X_2, respectively. The numerical integration of Eqn. 2.5 over X_1 and X_2 is performed over the two-dimensional grid of quadrature points specified. Line 10 is the correlation coefficient, ρ_{X_1,X_2}. Line 11 gives `idist_s`, the distribution index which defines the distribution of $S|X_1, X_2$, from among any of the two-parameter distributions in CYCLES 2.0 or the three-parameter, quadratic Weibull model. (Note that the two-parameter forms are fit to only two moments. Hence the third moment μ_3 in Eqn. 2.1 is used only when the three-parameter, quadratic Weibull model is selected.)

Line 12 is an integer variable: if this value is 0 then p_f is not estimated; G(X) is only evaluated at the median values of the random variables, to give a rough estimate of the median value of the safety margin. If this integer is non-zero then the full fatigue reliability analysis is performed. Line 13 is another integer variable ISIGMA. ISIGMA=0 indicates that the the second stress moment, μ_2 in Eqn. 2.1, is to be interpreted as the stress coefficient of variation; otherwise μ_2 is assumed to be the standard deviation of the stress range S. Lines 14 through 17 specify the bounding values for the fitted stress moments. Note that line 15 specifies an upper bound for the stress COV even though ISIGMA$\neq$0.

Lines 18 through 41 are the required input for the 24 random variables used in the formulation of fatigue damage and hence component lifetime. There is 1 line of input for each random variable. This line is of the format

```
idist  param_1  param_2 .... param_n
```

where n is the number of parameters required to define the distribution type `idist` (see table of distribution types). The program has the capability to model each random variable with one of twenty different probability distributions, as described in Table 4.1.

The ordering of the random variables in the input file follows the same sequence used in the description of the fatigue formulation from an earlier chapter. First are the four parameters $\overline{X_1}$, V_{X_1}, $\overline{X_2}$, V_{X_2}, of the two-parameter distributions describing the long term load variables, X_1 and X_2. These are followed by 12 variables that define the power-law fits of the stress mean, standard deviation (or COV, if ISIGMA=0), skewness and the stress cycles rate conditional on the environment variables, X_1 and X_2. The first three—$a0_{m1}$, $a1_{m1}$, and $a2_{m2}$—are used with the fixed parameters $x_{1,\text{ref}}$ and $x_{2,\text{ref}}$ to compute the mean stress. Similarly, the other three sets of parameters are used to find the stress COV, skewness and stress cycle rate. The stress is scaled by the stress concentration factor SCF (with Goodman correction applied using SM and SU) to get the stress at the fatigue-sensitive detail. The three stress moments—mean, standard deviation and skewness—are then used to define the conditional distribution for stress. Note again that only the first two moments are used if a two-parameter distribution type is specified by the stress amplitudes.

The fatigue properties C and b, which define the S–N relation, are entered next followed by the mean and ultimate stress used for the Goodman mean stress correction. The 22^{nd}, 23^{rd}, and 24^{th} random variables scale the accumulated fatigue damage and convert it to a failure time. The final variable is the target lifetime used to define the failure state function, Eqn. 3.1.

Line 42 specifies the number of lines below that specify the degree of correlation between any pair of the above 24 random variables. The value of the correlation

Table 4.1: Distribution Types available in CYCLES2

Dist#	Distribution Type	Parameter Values			
		#1	#2	#3	#4
1	Normal	Mean	Std. Dev.[a]		
2	Log Normal	Mean	Std. Dev.		
3	Exponential	Mean			
4	Weibull	Mean	Std. Dev.		
5	Gumbel	Mean	Std. Dev.		
6	Shifted Exponential	Mean	Dummy[b]	Shift	
7	Shifted Weibull	Mean	Std. Dev.	Shift	
8	Quadratic Weibull	Mean	Std. Dev.	Skewness	
9	Shifted Quad. Weib.	Mean	Std. Dev.	Skewness	Shift
10	4-moment Hermite	Mean	Std. Dev.	Skewness	Kurtosis
21	Normal	Mean	COV[c]		
22	Log Normal	Mean	COV		
23	Exponential	Mean			
24	Weibull	Mean	COV		
25	Gumbel	Mean	COV		
26	Shifted Exponential	Mean	Dummy	Shift	
27	Shifted Weibull	Mean	COV	Shift	
28	Quadratic Weibull	Mean	COV	Skewness	
29	Shifted Quad. Weib.	Mean	COV	Skewness	Shift
30	4-moment Hermite	Mean	COV.	Skewness	Kurtosis

[a]Std. Dev. denotes the standard deviation

[b]A dummy real real number is needed here simply for program consistency in reading the input

[c]COV denotes the coefficient of variation

coefficient is between -1 and 1 and represents the degree of linear relationship between any two variables. The input format of the correlation information on each line is

$\mathrm{RV1}_{\mathrm{index}} \quad \mathrm{RV2}_{\mathrm{index}} \quad \rho_{\mathrm{RV1,RV2}}$

For example, the correlation entry `5 8 -.63` indicates that the fifth and the eighth random variables have a correlation of -63%. Only the non-zero correlation values need to be specified.

The last four lines of input (lines 46 through 49) are four additional program control parameters. NPRI, line 46, is a print flag for the FORM algorithm. Permissible values of NPRI are 0, 1, and 2, in order of increasing output. RELAX, line 47, can be used to enforce slower time steps in the FORM algorithm by setting it equal to a number greater than 0. This may be useful if the ordinary FORM gradient search ("approximation" step in Section 2.2) fails to converge. IFORM, line 48, is used to invoke the simulation option of the program. When set to any number other than zero it performs Monte Carlo simulation until IFORM number of failures have occurred. ISTART, line 49, sets the initial value of the U vector at which the FORM gradient search begins. This is commonly taken to be the origin. Again, the user may seek to modify this if convergence problems arise.

4.3 Description of Output

The results of the program calculations consist of four parts:

1. Echo of program input
2. FORM/SORM results: failure probabilities, reliability indices, and importance factors

The next page shows the listing of the output from CYCLES corresponding to the sample input file given here. The echo of program input is printed in the same sequence in which the input was read. First are the program control parameters; i.e., the number of random variables and parameters used in the formulation. There are 14 parameters used in the current implementation. A brief summary of the 24 random variables follows, which includes the user-selected distribution types and their assigned parameters. Note that the output includes not only the input parameters for each variable, but also the internally computed output distribution parameters that are used throughout the execution of the program. The non-zero correlation coefficients of the physical variables (input values) are echoed, together with the internally computed normal U space correlation coefficients corresponding the above non-zero correlation coefficients (See "transformation" step discussed in Section 5). Finally the program control parameters NPRI, RELAX, IFORM, and ISTART are printed. Be aware that when the simulation option is invoked only a probability of failure is computed and printed. The remainder of the output described here results only when the FORM option is selected (IFORM = 0).

The results of the reliability analysis are the next items in the output file. A representative value of the excess life beyond the target life, T_t, is printed first. This value is obtained by substituting representative values (strictly, median values) for all random variables into equation 8. This representative excess life, reported in units of years, can be very useful when the CYCLES program is initially applied to a specific-fatigue problem. An unrealistically small or negative value may suggest that some of the input parameters are incorrectly defined, or at least that the specific algorithms of FORM/SORM, which are best-suited to rare failure events, may not apply.

Following the excess lifetime are the details of the FORM/SORM solutions for the failure probability. This includes the distance from the origin to the design point (known as BETA1 for FORM, BETA2 for SORM), curvatures at the design point, and the respective probabilities. The importance factors (squares of the direction cosines of the design point vector) are also printed for each random variable.

The final section of the program output are the result indicating the quadrature points of X_1 and X_2 for which the maximum fatigue damage was incurred. The corresponding the stress moments and the stress cycle rate at this quadrature points are also indicated.

Figures 4.1 and 4.2 show flow charts for the CYCLES computer code. The flow chart in Figure 4.1 corresponds to the overall execution of the program. Figure 4.2 provides details of the initial subroutine INPUT, which provides initial processing of the input random variables to obtain needed distribution parameters for subsequent

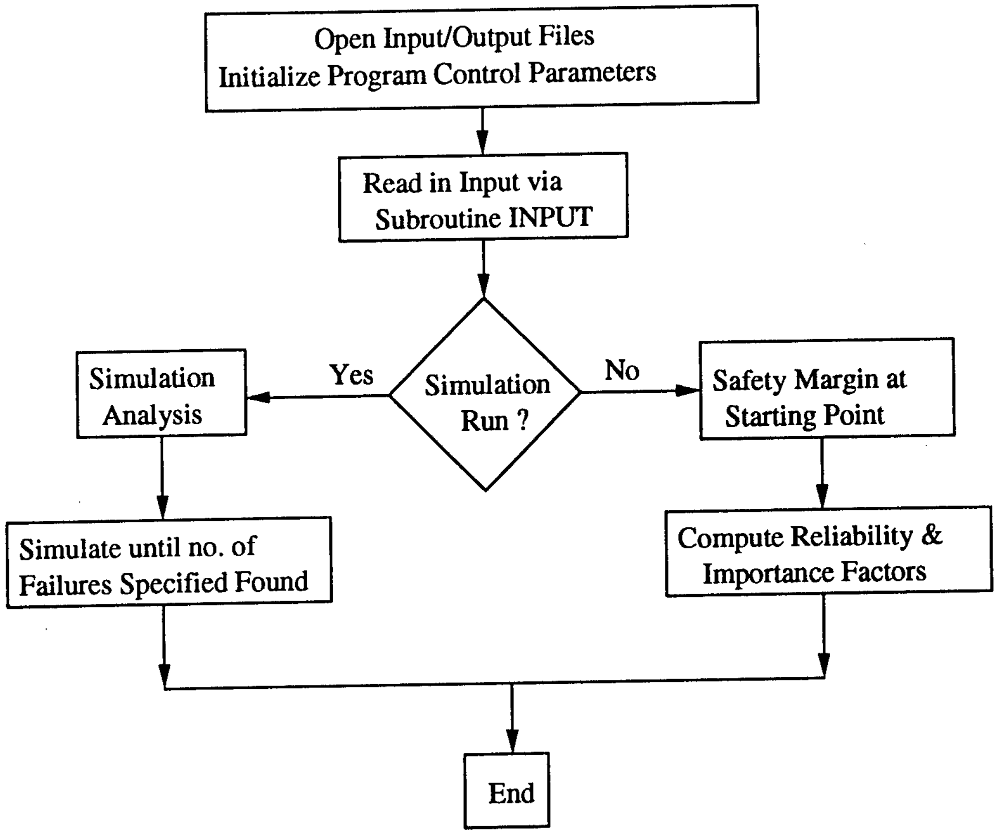


Figure 4.1: Flow Chart for Main Calling Program FORM

calculations.

4.4 Algorithm for Subroutine GRENZ

The following describes the algorithm for subroutine GRENZ to evaluate the G-function in Eqn. 3.1

- The 24 physical random variables, $Y_1 ... Y_{24}$, are related to a corresponding set $U_1 ... U_{24}$ of standard normal variables U_i. The Y_i include all the necessary variables for the fatigue life calculation except $E[S^b|X_1, X_2]$.
- The mean damage per unit time, $f_{avg}E[S^b]$, is required over a set of pairs (x_1, x_2) of environmental variables (Eqn. 2.10). This is obtained as follows:

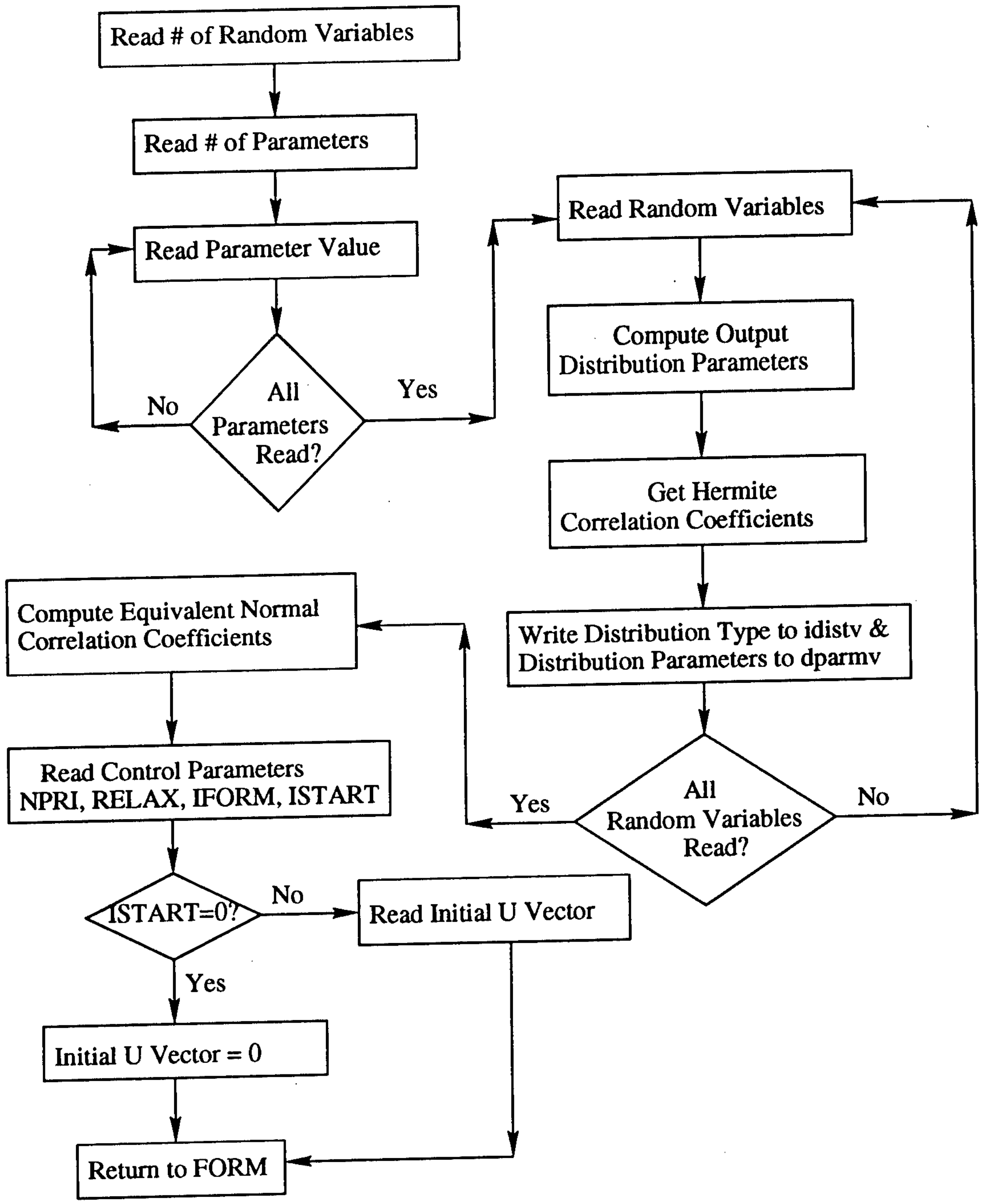


Figure 4.2: Flow Chart Subroutine INPUT

  - Given an x_1, x_2 pair, find the stress statistics—mean, COV (or standard deviation) , skewness, and cycle rate—from the power-law fits (Eqn. 2.1 and Eqn. 2.3).
  - From these moments and the assumed distribution type of S, find the required distribution parameters for $S|X_1, X_2$
  - Using these distribution parameters for $S|X_1, X_2$, estimate $E[S^b|X_1, X_2]$
- Once $f_{avg}E[S^b]$ has been obtained over all quadrature points, evaluate the mean damage $\overline{D}$ from Eqn. 2.10, the target life T_f from Eqn. 2.8, and the safety margin M (the output value of GRENZ) from Eqn. 3.1.

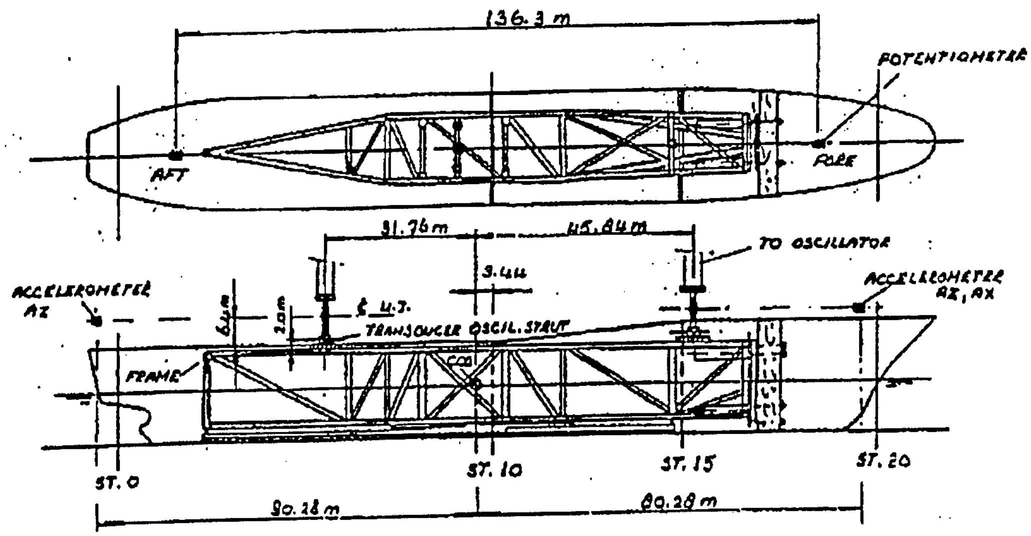
Chapter 5

Application to Ship Fatigue

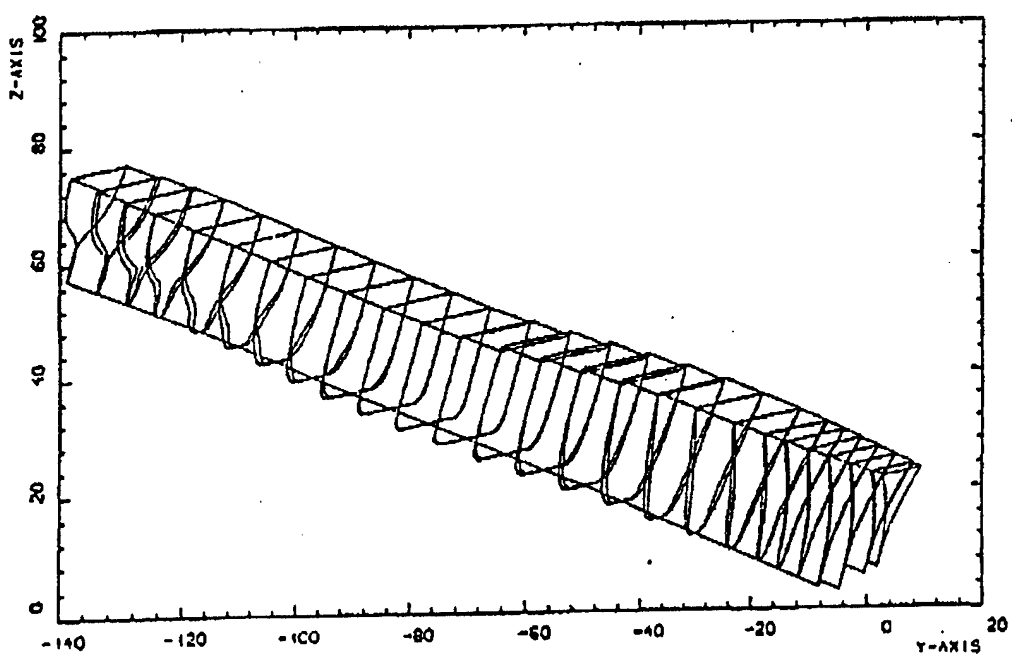
As noted earlier, the fatigue formulation established here may be of use in various applications; e.g., fatigue of aircraft, wind turbines, offshore structures, highway bridges under vehicle loads, and so forth. In this section we show a specific application to a fatigue-sensitive component of a marine vessel. An alternative application to wind turbines is shown in the following section.

A body plan and a strip model of the ship are shown in Figure 5.1 and the main particulars of the ship are given in Table 5.1. The cross-section of the ship changes along the length of the ship, with flared cross-sections at the ends of ship and box cross-sections towards mid-ship (see Fig. 5.1b). A ship moving in the waves is subjected to many kinds of loads: vertical and horizontal bending moments, torsional moments, side shell intermittent water pressures, etc. In this example we consider only the mid-ship vertical bending moments, and resulting mid-ship bending stresses, as loads on the fatigue-sensitive ship component. The sagging condition causes tensile stresses in the ship bottom. More details on this example can be found in References [4, 5].

We first review the underlying assumptions and numerical values used to model (1) the load environment, (2) the stress response, and (3) the resulting fatigue damage accumulation for a typical component of a marine vessel.



(a) Body Plan



(b) Strip Model

Figure 5.1: Model of monohull ship that will be analyzed using strip theory

Table 5.1: Main Particulars of Ship presented in Fig. 5.1

Specification	Value
Length between perpendiculars	166m
Beam	24.65m
Draught	8.85m
Weight	2×10^5 kN
Waterplane Area	2.84×10^3 sq.m

5.1 Load Environment

We assume that the long-term environment can be characterized by one environment variable X_1. This could, for example, be the significant wave height H_S, describing the steady-state ocean conditions over a 1–6 hour seastate. A distribution of X_1 should then be chosen to describe the long-term variation of the climate along the ship route (e.g., [1]). In this study, however, we instead choose the individual peak-to-trough wave height, H, as the environment variable X_1, and describe it by a long-term Weibull distribution. Note here that H is a local wave height, defined as the distance from the minimum wave surface elevation to the maximum elevation within each wave cycle. With this definition, a wave cycle is described by the wave surface between two mean upcrossings. Note that since we are not using the second environment variable X_2 in this example, we have made `nquadx2 = 0` in the input file.

In a short-term seastate with given H_S, we assume H to have Forristall distribution (see [3]). Note that the ship fatigue analysis studies [4] have suggested that the Forristall model predicts the simulated wave heights fairly accurately. This short-term distribution function is given as

$$F_{H|H_S}(h|h_S) = \text{Prob.}[H \leq h] = 1 - \exp\left[-\frac{(h/\sigma_\eta)^{2.126}}{8.42}\right] \tag{5.1}$$

in which $\sigma_\eta = H_S/4$. The long-term distribution function $F_{LT}(h)$ of wave heights can be found from the conditional Forristall distribution, $F_{H|H_S}$, as

$$F_{LT}(h) = \int_{h_S=0}^{\infty} F_{H|H_S}(h|h_S) f_{H_S}(h_S)\, dh_S \tag{5.2}$$

in which $f_{H_S}(h_S)$ is the long-term distribution of H_S. To demonstrate the methodology in this study, we assume H_S to have a Weibull distribution with mean $E[H_S] = 3$ meters and variance $\text{Var}[H_S] = 3.6 \text{ m}^2$ [7]. The resulting long-term distribution $F_{LT}(h)$, which can be evaluated numerically from Eqn. 5.2, is in turn also approximated by a two-parameter Weibull distribution form. The two parameters of this Weibull distribution are calibrated to preserve the first two moments $E[H_{LT}]$ and $E[H_{LT}^2]$ of the long-term wave heights. These moments can readily be found from the conditional distribution as

$$E[H_{LT}] = E_{H_S}[E[H|H_S]]; \quad E[H_{LT}^2] = E_{H_S}[E[H^2|H_S]] \tag{5.3}$$

where $E_{H_S}[]$ indicates taking expectation of random variable H_S. From these calculations we find the mean and the coefficient of variation (COV) of the long-term wave heights to be:

$$E[H_{LT}] = 1.81\text{meters} \qquad \text{COV}[H_{LT}] = 0.857$$

5.2 Gross Response

For this example, we use the nonlinear time domain analysis program NV1418 [2] to find the stresses in random wave conditions. We select the seastate described by $H_S = 5$m and $T_P = 10$s to analyze the ship response. Note that this is the most damaging seastate according to a linear analysis [4]. For all wave heights in this one hour seastate we find the corresponding sag bending moments in the response history. A regression analysis was performed to fit the following functional form:

$$E[\text{BM}|H] = aH^p \tag{5.4}$$

It is common to take the logarithm of this relation, leading to a linear regression problem for p and $\ln a$. This is consistent with the view that the scatter (conditional standard deviation) in bending moments is constant on log-log scale. (This is roughly equivalent to the assumption that the bending moments BM, given different heights H, have constant coefficients of variation.)

Table 5.2: Estimated mean and standard deviation of the regression parameters for bending moments (kN.m) given wave heights. The bending moments have been divided by 10^5.

Parameter	Value
a	0.453
σ_a	0.0284
p	1.168
σ_p	0.0404
ρ_{ap}	–0.972
$\sigma_{S\|H}$	0.746

We find here, however, that the scatter in BM is more nearly constant on the original linear scale; therefore, we seek to estimate a and p to minimize the sum-of-squared deviations $\sum_i [BM_i - aH_i^p]^2$. Because a and p enter this sum in a nonlinear fashion, their estimation requires nonlinear least-squares regression. For this purpose we use the Levenberg-Marquardt method [9], as implemented in Gnuplot [11]. This yields estimates of the parameters a, p, their standard errors σ_a, σ_p of estimation, and the correlation ρ_{ap} between their estimates for this data set. These standard errors reflect the uncertainty in the estimated parameters due to limited data. The resulting parameters are shown in Table 5.2. Finally, we assume that the bending moment can be converted to stresses by simply dividing by an appropriate section modulus, here taken as 35 m^3.

5.3 Numerical Values for Random Variables in Fatigue Analysis

The input random variables in the example fatigue analysis and their values are given in Table 5.3. The COV values in Table 5.3 should generally reflect the uncertainty in the parameters either due to limited data or due to lack of knowledge. The parameters relating stresses to wave heights are given in Table 5.2. To calibrate the median time

Table 5.3: Numerical values of means and COVs of random variables and their distribution types used in fatigue formulation. These are common to all three stresses: sag, hog and range.

Variable	Mean	COV	Dist.Type	Description
$E[X_1]$	1.81 (m)	0.05	Normal	Mean of Long-term H
COV$[X_1]$	0.857	0.1	Normal	COV of long-term H
f_0	0.1 (Hz)	0.2	Normal	Stress cycle rate
SCF	2.5	0.1	Normal	Stress concentration factor
C	2.4×10^{15}	0.5	Weibull	S–N factor
b	4	0	–	S–N exponent
Δ	0.02084	0.1	Normal	Damage threshold

to fail $\check{T}_f$ to a desired lifetime, we calibrate the mean value of Δ so as to have $\check{T}_f = 200$.

The example input and output files are presented in Appendix B. We find the CYCLES analysis yields a failure probability of about 1.2% for a target lifetime of 20 years. The relative importance of the random variables shows that the most important variable is the S–N factor C with about 77% importance. The next most important factors are found to be the COV of the wave heights and the stress concentration factor, followed by the mean wave height. Note that for the grid points specified, the most damaging wave height is about 8.3 meters.

Chapter 6

Wind Turbine Application

A primary incentive for the development of the CYCLES fatigue reliability program has been the fatigue problems facing the power industry with their wind turbine components over the past several years. In particular we consider here the Advanced Wind Turbines' AWT–26 P2 prototype in Tehachapi, California for which data were collected in 1994. This is a turbine with a downwind, two-bladed, free-yaw machine, with 26 m diameter teetered rotor, and power rating of 275 kW [8, 10]. They are perhaps a typical example of data collected on prototype turbines during development efforts around the world. These data are from a single location on the turbine—the blade root flatwise bending—but could be from any component of loading with fatigue damaging potential. The data consist of over thirty hours of turbine operation collected in ten minute segments.

Table 6.1 shows the number of ten minute samples that fall into each wind bin divided over both wind speed and turbulence intensity, defined as standard deviation of wind speed divided by mean wind speed. Wind speed runs from about 5 to 20 m/s and turbulence intensity ranges from about 8 to 30%, although most of the samples fall on the lower half of that range.

The moments of the rainflow-range amplitudes were calculated for all the 30-plus hours of data. Figures 6.1, 6.2 and 6.3 show the results for the mean, COV, and skewness, respectively, across all the bins in Table 6.1. There appears to be an upward, approximately linear trend of the mean with wind speed, a mild tendency for COV to decrease with wind speed, and no particular trend of skewness with wind

Table 6.1: Number of 10-minute samples as a function of mean wind speed $\overline{V}$ (m/s) and turbulence intensity I for the 10-minute samples

$\overline{V}$	Turbulence Intensity I							
	0.095	0.125	0.155	0.185	0.215	0.245	0.275	0.305
5.48	0	0	0	2	3	1	0	1
7.49	0	0	1	14	5	1	0	0
9.50	0	0	10	17	5	0	0	0
11.51	5	7	31	16	5	2	0	0
13.52	8	9	13	7	3	0	0	0
15.53	0	2	3	0	0	0	0	0
17.54	3	9	0	0	0	0	0	0
19.56	4	4	0	0	0	0	0	0

speed.

Moment behavior as a function of wind conditions is illustrated by a standard regression fit of the moment data over the two dimensional space of wind speed, V ($=X_1$), and turbulence intensity, I ($=X_2$), with the following functional form.

$$\mu_i = a_{0i}\left(\frac{x_1}{x_{1_{\mathrm{ref}}}}\right)^{a_{1i}}\left(\frac{x_2}{x_{2_{\mathrm{ref}}}}\right)^{a_{2i}} \tag{6.1}$$

where μ_i, for $i = 1, 2, 3$ refer to the mean, standard deviation (or COV), and skewness of the stresses, respectively. $X_{1,\mathrm{ref}}$ and $X_{2,\mathrm{ref}}$ are the reference values for the independent variables x_1 and x_2, respectively. We choose the reference values as the geometric mean values found from the data; for example, in terms of the individual mean wind speeds, V_i, observed in each 10-minute segment. In this example $X_{1,\mathrm{ref}}$=11.35m/s, and the analogous geometric mean of the turbulence intensity is $X_{2,\mathrm{ref}}$=0.157. By using these geometric means to normalize our fit, we achieve uncorrelated estimates of the parameters a_{0i}, a_{1i}, and a_{2i}. Note that we do not bin the 10-minute results to perform a weighted regression analysis as was done in Reference [10]. Rather each of the 10-minute outcomes (wind speed and turbulence intensity) and the corresponding load moments are treated individually in the regression analysis. This results in slight

Table 6.2: Regression parameters for load amplitude moments vs. wind speed V and turbulence intensity I

Moment	Parameter	Mean	Sigma
Mean (μ_1)	a_{01}	0.253	0.003
	a_{11}	0.974	0.053
	a_{21}	0.346	0.060
COV (μ_2)	a_{02}	1.117	0.004
	a_{12}	-0.190	0.017
	a_{22}	0.163	0.020
Skewness (μ_3)	a_{03}	1.788	0.013
	a_{13}	-0.201	0.038
	a_{23}	-0.005	0.043
Cycle Rate (f_{avg})	f_0	6.635	0.022
	f_1	0.191	0.017
	f_2	-0.096	0.019

differences in the estimated mean and standard deviation of the regression parameters as found here (see Table 6.2) versus Reference [10].

The rate at which cycles are accumulated is also an important quantity in conducting a fatigue analysis. The cycle rate can be treated just like the moments of the load amplitudes in the previous section. Figure 6.4 shows the AWT cycle rate data plotted versus wind speed. Again, for this response quantity, there is minimal dependence on I, and significant dependence on V. However, the relative size of the change in cycle rate with wind speed is small enough ($\pm 15\%$) that variations in the rate will have a minimal effect on lifetime estimates.

As described in the general fatigue formulation there are three distinct aspects of the fatigue perfomance of a structure. They are the characterization of the load, the structural response to that load, and a failure criterion which in this case is analogous to the fatigue properties of the material being used. The details of the site wind conditions, the power-law fit and the $S-N$ material properties can be found,

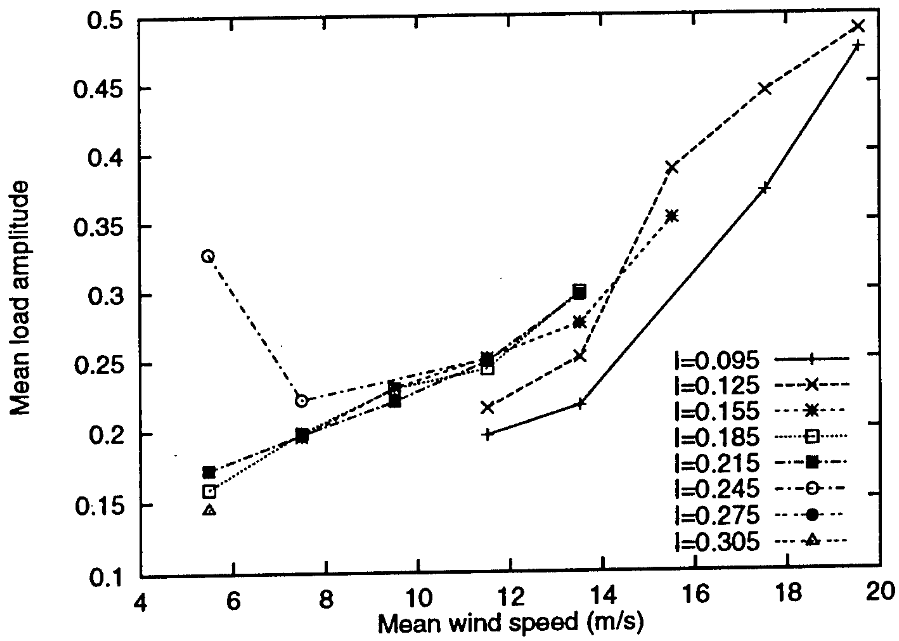


Figure 6.1: First moment (mean load amplitude) from the AWT data set

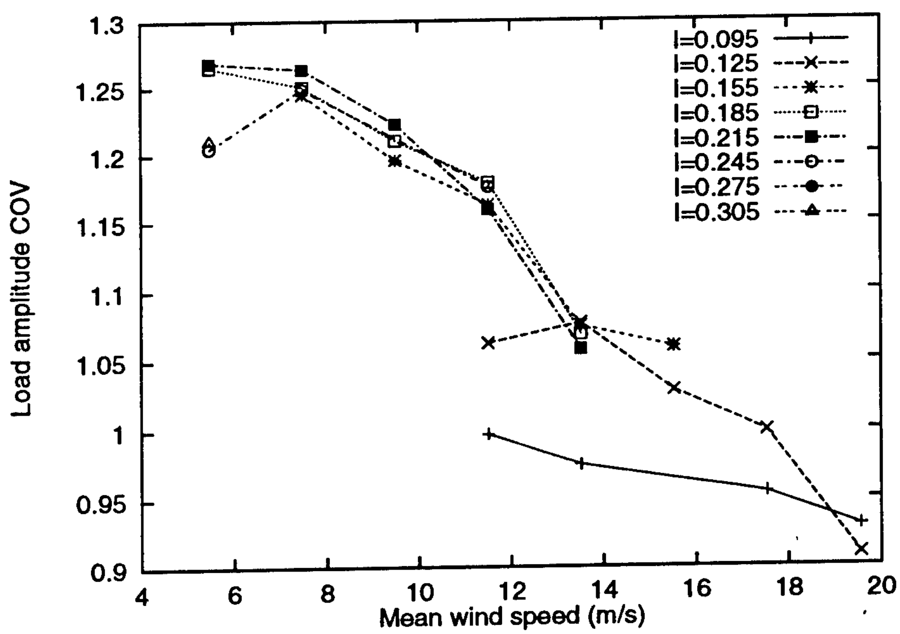


Figure 6.2: Second moment (load amplitude COV) measurement from the AWT data set

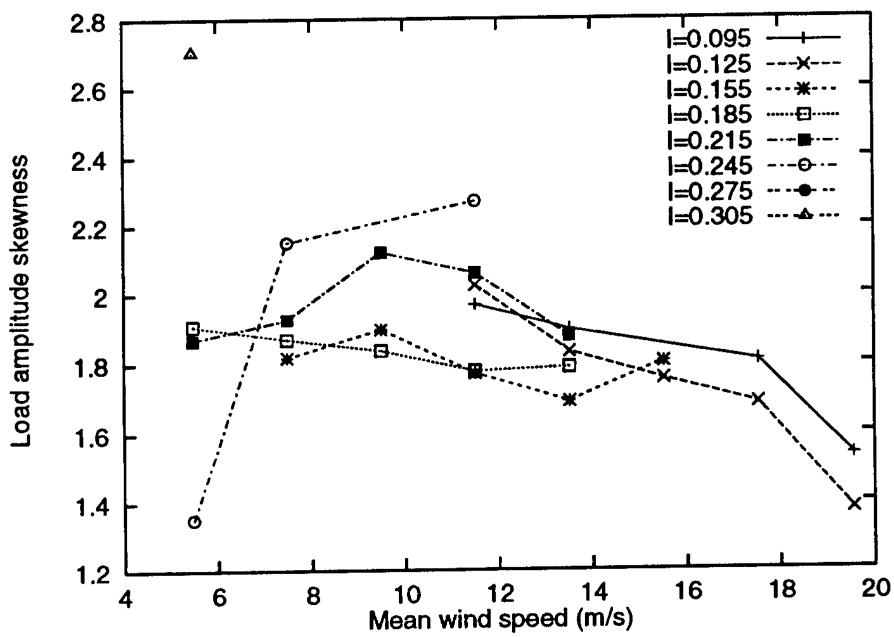


Figure 6.3: Third moment (load amplitude skewness) measurement from the AWT data set

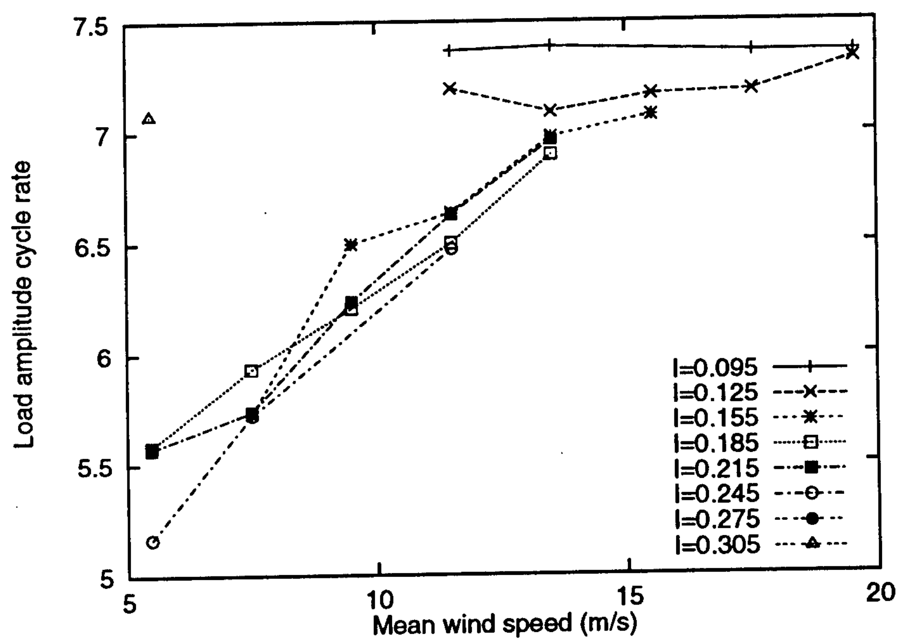


Figure 6.4: Load cycle rate measurement from the AWT data set

for example, in References [6, 10].

The input file for this example is provided in Appendix A and the remaining parameters for the reliability formulation can be found in this input file.

The output from the CYCLES analysis for the wind turbine is also given in Appendix A. This example results show a probability of failure of approximately 1.1% for a target lifetime of 20 years with a median lifetime of 508 years. Of equal interest is the relative importance of each random variable on the fatigue life of the turbine. Results show that the leading coefficient in the S-N relationship is the most important source of variability, supplying about 85% of the total variability in this example. The turbulence intensity is next with approximately 10% contribution, followed by the mean wind speed, the COV of the turbulence intensity, and the turbulence intensity exponents for the mean stress and stress COV power-law fits. The relative importance of each random variable provides valuable insight to the designer who is attempting to reduce the effects of fatigue damage.

List of References

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Appendix A

Sample Input and Output Files for Wind Example

Sample Input File from Wind Example: **cycles.in**

```
Input Values                        Descriptive Comments
-------------          ------------------------------------------------
4  7  5  6       ;      IOIN,IOOUT,NINTER,NOUTER
24               ;      NVAR
14               ;      NPARM
11.351           ; 1    X1REF:          Fixed Parameters  1...NPARM
 0.157           ; 2    X2REF:
4                ; 3    idistx1
10               ; 4    nquadx1
2                ; 5    idistx2
10               ; 6    nquadx2
0.0              ; 7    Correlation (X1, X2)
2                ; 8    idist response given X1,X2 (any 2-param dist.+QWeib)
1                ; 9    =0: Evaluate G-Function once at mean values of param.s
0                ; 10   isigma=0 -> stress moment m2 is COV, else m2 is std.dev
10.0             ; 11   Max mean stress
 1.5             ; 12   Max stress COV
 3.0             ; 13   Max stress skewness
-0.5             ; 14   Min stress skewness                     1...NVAR
21  7.5        0.075       ; 1    X1AVG:   N(mu,cov)   Random Variables
24  0.56       0.075       ; 2    X1COV:   W(mu,cov)
21  0.15       0.075       ; 3    X2AVG:   N(mu,cov)
24  0.22       0.075       ; 4    X2COV:   W(mu,cov)
1   0.253      0.003       ; 5    a0_m1:   N(mu,sig) stress mean
1   0.974      0.053       ; 6    a1_m1:   N(mu,sig)
1   0.346      0.060       ; 7    a2_m1:   N(mu,sig)
1   1.117      0.004       ; 8    a0_m2:   N(mu,sig) stress COV
1  -0.190      0.017       ; 9    a1_m2:   N(mu,sig)
1   0.163      0.020       ; 10   a2_m2:   N(mu,sig)
1   1.788      0.013       ; 11   a0_m3:   N(mu,sig) stress skewness
1  -0.201      0.038       ; 12   a1_m3:   N(mu,sig)
1  -0.005      0.043       ; 13   a2_m3:   N(mu,sig)
1   6.635      0.022       ; 14   a0_f0:   N(mu,cov) stress cycle rate
1   0.191      0.017       ; 15   a1_f0:   N(mu,sig)
1  -0.096      0.019       ; 16   a2_f0:   N(mu,sig)
21  1.         0.          ; 17   SCF:     N(mu,cov)
24  2.420D16   0.7         ; 18   C:       W(mu,cov)
21  8.         0.          ; 19   B:       N(mu,cov)
1   0.         0.          ; 20   SM:      N(mu,sig)
21  85.0       0.          ; 21   SU:      N(mu,cov)
21  1.         0.          ; 22   DELTA:   N(mu,cov)
21  1.         0.          ; 23   AVAIL:   N(mu,cov)
21  20.        0.          ; 24   TARLF:   N(mu,cov)
3                          ; Number of lines containing correlation coeff.
5  8  -.63                 ; Correlation Coefficients
5  11 -.64                 ; Correlation Coefficients
8  11  .34                 ; Correlation Coefficients
0                ;    NPRI   (FORM printing index)
0                ;    RELAX  (=0 for ordinary FORM; >0. for slower steps)
0                ;    IFORM  (=0 for FORM, else simulate IFORM failures)
0                ;    ISTART (=0 Start at mean; else read in starting vector)
```

Sample Output File from Wind Example: **cycles.out**

```
NUMBER OF RANDOM VARIABLES:          24
     NUMBER OF PARAMETERS:           14

          Parameter       Value
          ======================
              1           11.35
              2          0.1570
              3           4.000
              4           10.00
              5           2.000
              6           10.00
              7              0.
              8           2.000
              9           1.000
             10              0.
             11           10.00
             12           1.500
             13           3.000
             14         -0.5000

Variable #  Dist #   Parm # 1    Parm # 2    Parm # 3    Parm # 4
=================================================================
X1AVG         21       7.500    0.7500E-01     0.          0.
                       7.500    0.5625         0.          0.
X1COV         24      0.5600    0.7500E-01     0.          0.
                      0.5600    0.4200E-01   16.41      0.5783
X2AVG         21      0.1500    0.7500E-01     0.          0.
                      0.1500    0.1125E-01     0.          0.
X2COV         24      0.2200    0.7500E-01     0.          0.
                      0.2200    0.1650E-01   16.41      0.2272
a0_m1          1      0.2530    0.3000E-02     0.          0.
                      0.2530    0.3000E-02     0.          0.
a1_m1          1      0.9740    0.5300E-01     0.          0.
                      0.9740    0.5300E-01     0.          0.
a2_m1          1      0.3460    0.6000E-01     0.          0.
                      0.3460    0.6000E-01     0.          0.
a0_m2          1       1.117    0.4000E-02     0.          0.
                       1.117    0.4000E-02     0.          0.
a1_m2          1     -0.1900    0.1700E-01     0.          0.
                     -0.1900    0.1700E-01     0.          0.
a2_m2          1      0.1630    0.2000E-01     0.          0.
                      0.1630    0.2000E-01     0.          0.
a0_m3          1       1.788    0.1300E-01     0.          0.
                       1.788    0.1300E-01     0.          0.
a1_m3          1     -0.2010    0.3800E-01     0.          0.
                     -0.2010    0.3800E-01     0.          0.
a2_m3          1   -0.5000E-02  0.4300E-01     0.          0.
                   -0.5000E-02  0.4300E-01     0.          0.
```

a0_f0	1	6.635	0.2200E-01	0.	0.
		6.635	0.2200E-01	0.	0.
a1_f0	1	0.1910	0.1700E-01	0.	0.
		0.1910	0.1700E-01	0.	0.
a2_f0	1	-0.9600E-01	0.1900E-01	0.	0.
		-0.9600E-01	0.1900E-01	0.	0.
SCF	21	1.000	0.	0.	0.
		1.000	0.	0.	0.
C	24	0.2420E+17	0.7000	0.	0.
		0.2420E+17	0.1694E+17	1.451	0.2669E+17
B	21	8.000	0.	0.	0.
		8.000	0.	0.	0.
SM	1	0.	0.	0.	0.
		0.	0.	0.	0.
SU	21	85.00	0.	0.	0.
		85.00	0.	0.	0.
DELTA	21	1.000	0.	0.	0.
		1.000	0.	0.	0.
AVAIL	21	1.000	0.	0.	0.
		1.000	0.	0.	0.
TARLF	21	20.00	0.	0.	0.
		20.00	0.	0.	0.

```
** CORR [ Xi, Xj ] = CORR MATRIX OF PHYSICAL VARIABLES **
RX  8 - RX  5 = -0.6300
RX 11 - RX  5 = -0.6400
RX 11 - RX  8 =  0.3400

** CORR [ Ui, Uj ] = CORR MATRIX OF GAUSSIAN VARIABLES **

RX  8 - RX  5 = -0.6300
RX 11 - RX  5 = -0.6400
RX 11 - RX  8 =  0.3400


 FORM PRINT INDEX (0,1, or 2):      0.
 FORM PARAMETER (0=normal)  :       0.
 NUM OF SIMULATIONS? (0=FORM):      0.
  START FORM AT MEAN? (0=yes):      0.

safety margin at starting point:   0.48855E+03

                        e[s^b]:   0.129292E+07


      **  beta   failure prob **

form:     2.325   0.1005E-01
sorm:     2.294   0.1091E-01
```

```
number of iterations:      7


DESIGN POINT
============
BETA1 =     2.325
    I          X             U          AU        AU**2
X1AVG     7.581E+00    1.435E-01   -0.062     0.004            *.
X1COV     5.671E-01    3.840E-02   -0.017     0.000             .
X2AVG     1.582E-01    7.315E-01   -0.315     0.099          ***.
X2COV     2.258E-01    2.432E-01   -0.105     0.011            *.
a0_m1     2.531E-01    2.396E-02   -0.010     0.000             .
a1_m1     9.714E-01   -4.851E-02    0.021     0.000             .
a2_m1     3.602E-01    2.371E-01   -0.102     0.010            *.
a0_m2     1.117E+00    1.132E-01   -0.049     0.002             .
a1_m2    -1.914E-01   -8.391E-02    0.036     0.001             .
a2_m2     1.697E-01    3.359E-01   -0.145     0.021            *.
a0_m3     1.788E+00    0.000E+00    0.000     0.000             .
a1_m3    -2.010E-01    0.000E+00    0.000     0.000             .
a2_m3    -5.000E-03    0.000E+00    0.000     0.000             .
a0_f0     6.635E+00    4.061E-03   -0.002     0.000             .
a1_f0     1.910E-01   -1.948E-03    0.001     0.000             .
a2_f0    -9.582E-02    9.391E-03   -0.004     0.000             .
SCF       1.000E+00    0.000E+00    0.000     0.000             .
C         1.556E+15   -2.144E+00    0.922     0.850             .*********
B         8.000E+00    0.000E+00    0.000     0.000             .
SM        0.000E+00    0.000E+00    0.000     0.000             .
SU        8.500E+01    0.000E+00    0.000     0.000             .
DELTA     1.000E+00    0.000E+00    0.000     0.000             .
AVAIL     1.000E+00    0.000E+00    0.000     0.000             .
TARLF     2.000E+01    0.000E+00    0.000     0.000             .


    Maximum damage occured at X1 =     9.247
    Maximum damage occured at X2 =     0.214

    At this X1 & X2 moments of stress are:
        Mean stress            =     0.232
        Std. Dev. of stress    =     0.284
        Skewness of stress     =     1.860
        cycle rate of stress =       6.194
```

Appendix B

Sample Input and Output Files for Ship Example

Sample Input File: **cycles.in**

```
4  7  5  6    ;      IOIN,IOOUT,NINTER,NOUTER
24            ;      NVAR
14            ;      NPARM
1.0           ; 1    X1REF:        Fixed Parameters   1...NPARM
1.0           ; 2    X2REF:
4             ; 3    idistx1
10            ; 4    nquadx1
1             ; 5    idistx2
0             ; 6    nquadx2
0.0           ; 7    Correlation (X1, X2)
4             ; 8    idist response given X1,X2 (LN=2,22,W=4,24,else QW)
1             ; 9    =0: Evaluate G-Function once at mean values of param.s
1             ; 10   isigma=0 -> stress moment m2 is COV, else m2 is std.dev
 1.e20        ; 11   Max mean stress
 1.5          ; 12   Max stress COV
 1.e20        ; 13   Max stress skewness
-0.5          ; 14   Min stress skewness                         1...NVAR
21  1.81      0.05      ; 1    X1AVG:    N(mu,cov)   Random Variables
21  0.857     0.1       ; 2    X1COV:    W(mu,cov)
1   0.0       0.0       ; 3    X2AVG:    N(mu,cov)
1   0.0       0.0       ; 4    X2COV:    W(mu,cov)
21  1.294     0.06267   ; 5    a0_m1:    N(mu,sig) stress mean
1   1.168     0.0404    ; 6    a1_m1:    N(mu,sig)
1   0.0       0.0       ; 7    a2_m1:    N(mu,sig)
1   2.1314    0.        ; 8    a0_m2:    N(mu,sig) stress sigma
1   0.0       0.0       ; 9    a1_m2:    N(mu,sig)
1   0.0       0.0       ; 10   a2_m2:    N(mu,sig)
1   0.1       0.0       ; 11   a0_m3:    N(mu,sig) stress skewness
1   0.0       0.0       ; 12   a1_m3:    N(mu,sig)
1   0.0       0.0       ; 13   a2_m3:    N(mu,sig)
21  0.1       0.2       ; 14   a0_f0:    N(mu,cov) stress cycle rate
1   0.0       0.0       ; 15   a1_f0:    N(mu,sig)
1   0.0       0.0       ; 16   a2_f0:    N(mu,sig)
21  2.5       0.1       ; 17   SCF:      N(mu,cov)
24  2.4D15    0.5       ; 18   C:        W(mu,cov)
21  4.        0.        ; 19   B:        N(mu,cov)
1   0.        0.        ; 20   SM:       N(mu,sig)
21  85.0      0.        ; 21   SU:       N(mu,cov)
21  0.02084   0.1       ; 22   DELTA:    N(mu,cov)
21  1.        0.        ; 23   AVAIL:    N(mu,cov)
21  20.       0.        ; 24   TARLF:    N(mu,cov)
1                       ; Number of lines containing correlation coeff.
5  6  -.97              ; Correlation Coefficients
0             ;      NPRI   (FORM printing index)
0             ;      RELAX  (=0 for ordinary FORM; >0. for slower steps)
0             ;      IFORM  (=0 for FORM, else simulate IFORM failures)
0             ;      ISTART (=0 Start at mean; else read in starting vector)
```

Sample Output File: **cycles.out**

```
NUMBER OF RANDOM VARIABLES:          24
      NUMBER OF PARAMETERS:          14
```

Parameter	Value
1	1.000
2	1.000
3	4.000
4	10.00
5	1.000
6	0.
7	0.
8	4.000
9	1.000
10	1.000
11	0.1000E+21
12	1.500
13	0.1000E+21
14	-0.5000

Variable #	Dist #	Parm # 1	Parm # 2	Parm # 3	Parm # 4
X1AVG	21	1.810	0.5000E-01	0.	0.
		1.810	0.9050E-01	0.	0.
X1COV	21	0.8570	0.1000	0.	0.
		0.8570	0.8570E-01	0.	0.
X2AVG	1	0.	0.	0.	0.
		0.	0.	0.	0.
X2COV	1	0.	0.	0.	0.
		0.	0.	0.	0.
a0_m1	21	1.294	0.6267E-01	0.	0.
		1.294	0.8109E-01	0.	0.
a1_m1	1	1.168	0.4040E-01	0.	0.
		1.168	0.4040E-01	0.	0.
a2_m1	1	0.	0.	0.	0.
		0.	0.	0.	0.
a0_m2	1	2.131	0.	0.	0.
		2.131	0.	0.	0.
a1_m2	1	0.	0.	0.	0.
		0.	0.	0.	0.
a2_m2	1	0.	0.	0.	0.
		0.	0.	0.	0.
a0_m3	1	0.1000	0.	0.	0.
		0.1000	0.	0.	0.
a1_m3	1	0.	0.	0.	0.
		0.	0.	0.	0.
a2_m3	1	0.	0.	0.	0.
		0.	0.	0.	0.

```
a0_f0        21    0.1000       0.2000       0.      0.
                   0.1000       0.2000E-01   0.      0.
a1_f0         1    0.           0.           0.      0.
                   0.           0.           0.      0.
a2_f0         1    0.           0.           0.      0.
                   0.           0.           0.      0.
SCF          21    2.500        0.1000       0.      0.
                   2.500        0.2500       0.      0.
C            24    0.2400E+16   0.5000       0.      0.
                   0.2400E+16   0.1200E+16   2.101   0.2710E+16
B            21    4.000        0.           0.      0.
                   4.000        0.           0.      0.
SM            1    0.           0.           0.      0.
                   0.           0.           0.      0.
SU           21    85.00        0.           0.      0.
                   85.00        0.           0.      0.
DELTA        21    0.2084E-01   0.1000       0.      0.
                   0.2084E-01   0.2084E-02   0.      0.
AVAIL        21    1.000        0.           0.      0.
                   1.000        0.           0.      0.
TARLF        21    20.00        0.           0.      0.
                   20.00        0.           0.      0.

** CORR [ Xi, Xj ] = CORR MATRIX OF PHYSICAL VARIABLES **
RX  6 - RX  5 = -0.9700

** CORR [ Ui, Uj ] = CORR MATRIX OF GAUSSIAN VARIABLES **

RX  6 - RX  5 = -0.9700


 FORM PRINT INDEX (0,1, or 2):       0.
 FORM PARAMETER (0=normal)   :       0.
 NUM OF SIMULATIONS? (0=FORM):       0.
  START FORM AT MEAN? (0=yes):       0.

safety margin at starting point:   0.17996E+03

                        e[s^b]:  0.192558E+03


      **  beta   failure prob **

form:    2.282   0.1123E-01
sorm:    2.256   0.1205E-01

number of iterations:      6


DESIGN POINT
```

```
============
BETA1 =     2.282
   I          X              U           AU        AU**2
X1AVG    1.842E+00     3.544E-01    -0.155     0.024                **.
X1COV    9.170E-01     6.998E-01    -0.307     0.094               ***.
X2AVG    0.000E+00     0.000E+00     0.000     0.000                  .
X2COV    0.000E+00     0.000E+00     0.000     0.000                  .
a0_m1    1.286E+00    -9.453E-02     0.041     0.002                  .
a1_m1    1.173E+00     1.209E-01    -0.053     0.003                 *.
a2_m1    0.000E+00     0.000E+00     0.000     0.000                  .
a0_m2    2.131E+00     0.000E+00     0.000     0.000                  .
a1_m2    0.000E+00     0.000E+00     0.000     0.000                  .
a2_m2    0.000E+00     0.000E+00     0.000     0.000                  .
a0_m3    1.000E-01     0.000E+00     0.000     0.000                  .
a1_m3    0.000E+00     0.000E+00     0.000     0.000                  .
a2_m3    0.000E+00     0.000E+00     0.000     0.000                  .
a0_f0    1.066E-01     3.278E-01    -0.144     0.021                 *.
a1_f0    0.000E+00     0.000E+00     0.000     0.000                  .
a2_f0    0.000E+00     0.000E+00     0.000     0.000                  .
SCF      2.664E+00     6.551E-01    -0.287     0.082               ***.
C        4.499E+14    -2.001E+00     0.876     0.768                  .*********
B        4.000E+00     0.000E+00     0.000     0.000                  .
SM       0.000E+00     0.000E+00     0.000     0.000                  .
SU       8.500E+01     0.000E+00     0.000     0.000                  .
DELTA    2.047E-02    -1.788E-01     0.078     0.006                  .*
AVAIL    1.000E+00     0.000E+00     0.000     0.000                  .
TARLF    2.000E+01     0.000E+00     0.000     0.000                  .


    Maximum damage occured at X1 =    8.374
    Maximum damage occured at X2 =    1.000

    At this X1 & X2 moments of stress are:
        Mean stress            =   15.555
        Std. Dev. of stress    =    2.131
        Skewness of stress     =    0.100
        cycle rate of stress   =    0.107
```

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14. ABSTRACT

This document describes a general fatigue formulation and its implementation into the (pre-existing) Cycles 2.0. The generalizations were particularly motivated for applications to fatigue-sensitive components of ships or of wind-turbine machines. The applications share the common feature of a complex load environment and hence the need to best utilize available loads data, which may be available from physical measurement or from simulation. Examples of both ship and wind-turbine applications are shown in this report.

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